PART I –

6. Let the elements of $S = \{10, 11, 12, . . . , 100\}$ be the perimeters of triangles with integral side lengths.

12. In racing over a given distance *d* yards, Abe beat Barbara by 20 yards, Barbara beat Chris by 10 yards, and Abe beat Chris by 28 yards. If each person ran at a constant speed, compute *d*.

(A) 100 (B) 120 (C) 150 (D) 200 (C) 240

13. There are two values of *m*

19. Let $z = a + bi$ where a and b are positive integers and i is the imaginary unit. Compute the smallest possible value of $a + b$ for which $z + z \quad + z$ is a real number.

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

20. There is one value of k for which the system of equations $r \quad + s \quad = t$ and $r + s + t = k$ will have exactly one real solution (r, s, t). What is this value of k?

(A) –3 (B) ! $\frac{3}{2}$ (C) –1 (D) $\frac{1}{2}$ (E) None of these

21. In the diagram, AB = 3, BC = 4, CD = 12, AD = 13, and $\overline{AB} \perp \overline{BC}$. Compute the ratio of the area of ΔABD to the area of ΔBCD.

(A) $\frac{3}{4}$ (B) $\frac{4}{5}$ (C) $\frac{5}{6}$ (D) $\frac{6}{7}$ (E) $\frac{7}{8}$

22. Dr. Garner presented a challenge to his math class. He wrote a set of consecutive positive integers, beginning with 1, on a piece(i) 0.2 (t) 0.2 (ng 0 00 276.72 602) TjET 19A 0.24 0 0 0.24 354.9

1. C Clearly, the smallest *prim-prime* is 2 + 3 = 5. The largest prim-

8. D Expanding $(3 + 4k)x + (2 - k)y = 3$ we obtain $3x + 4kx + 2y - ky = 3$. Rearranging and regrouping terms yields $3x + 2y + k(4x - y) = 3$. This will be true for all values of k if the equations $4x - y = 0$ and $3x + 2y = 3$ are both true. Solving these two equations simultaneously gives $(x, y) = (— \ —)$. Therefore, $x + y = —$.

9. E Represent the four-

14. E Since $\sin 3x = \cos(90 - 3x)$, $7x = 90 - 3x$ from which $10x = 90$, or more generally, $10x = 90 + 360k$ % $x = 9 + 36k$. Since $0° < x < 360°$, there will solutions for $k = 0, 1, 2, 3, \ldots, 9$ for a total of 10 solutions.
Also, $\sin 3x = \cos(270 + 3x)$, $7x = 270 + 3x$ from which $4x = 270$, or more generally, $4x = 270 + 360k$ % $x = 67.5 + 90k$. Thus there will be solutions for $k = 0, 1, 2, 3$, for an additional 4 solutions. Total number of solutions is 14.

15. C $374_b = 3b^2 + 7b + 4 = (3b+4)(b+1)$. Since $(3b+4) - 3(b+1) = 1$, $(3b+4)$ and $(b+1)$ have no common factors greater than 1 for all values of b. Therefore, the product will be a perfect square only if both 3b+4 and b+1 are perfect squares. Examining the first few integers for $b > 7$, we find that when $b = 15$, the factors become 49 and 16, both perfect squares, and $374_b = 28^2 = 784$ in base ten.

16. C Using the Pythagorean Theorem, –

19. E $z = a + bi$,

23. E Since + = is an integer, $xy = 10^k$ where k is a nonnegative integer. Since x and y are less than 100, $k < 4$.

If $k = 0$, (x, y) = (1, 1)

If $k = 1$, $xy = 10$. Since 10 has four factors, there are four ordered pairs, namely (1, 10), (2, 5), (5, 2), and (10, 1).

If $k = 2$, xy = 100 and there are seven ordered pairs, (2, 50), (4, 25), (5, 20), (10, 10), (20, 5), (25, 4), and (50, 2).

If $k = 3$, $xy = 1000$ and there are four ordered pairs, (20, 50), (25, 40), (40, 25), and (50, 20).

Therefore, altogether there are 1 + 4 + 7 +4 = 16 possible ordered pairs.

24. D Construct chord BC. Since minor arc AB measures 90°,
&ACB is