THE 2018 2019 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

90 MINUTES

1. Most of Harry Potter's classmates at Hogwarts School of Witchcraft and Wizardry plan to attend the school's annual Halloween Party. What is the minimum number of students who would have to attend in order to

6. The symbols $uv_{å}$ and $vy_{æ}$ represent two-digit numbers in bases *r* and *s* respectively.

15. The line $y = mx + 1$ ($m > 0$) intersects the ellipse $x^2 + 4y^2 = 1$ in exactly one point.
If is

21. In Dr. Garner's math class, the number of students receiving A's is at least one-fifth of the number of students receiving B's and at most one-sixth the number of students receiving C's. The number of students receiving A's or B's is at least 23. If A, B, and C are the only grades given in the class, what is the minimum number of students receiving C's?

(A) 18 (B) 20 (C) 22 (D) 23 (E) 24

22. In the diagram, a semicircle is inscribed in a quarter circle.

9. B In any equation of the form $x^3 + ax^2 + bx + c = 0$ with roots *p*, *q*, and *r*,

(i) $p + q + r = -a$, (ii) $pq + pr + qr = b$, and (iii) $pqr = -c$

The three roots of the given equation T^7 F t sT^6 E t xT E ?L r

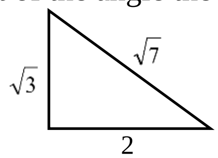
15. E Substituting $y = mx + 1$ into $x^2 + 4y^2 = 1$ and rearranging terms, we obtain

$$(4m^2+1)x^2 + 8mx + 3 = 0$$

In order for the two graphs to intersect exactly once, the discriminant must be zero.

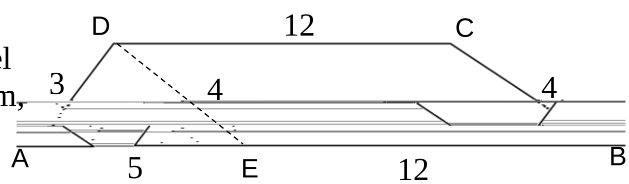
$$(8m)^2 - 4(4m^2+1)(3) = 16m^2 - 12 = 0 \text{ and } m = \frac{\sqrt{3}}{2}.$$

Since the slope of a line is the tangent of the angle the line makes with the positive x-axis, $\tan\theta = \frac{\sqrt{3}}{2}$ from which $\sin^2\theta = \frac{3}{7}$.



16. E Let a and b be, respectively, the number of pennies that Debbie and Don had in their stacks, and let x be the certain number of pennies. From the given information, we obtain the following two equations: $a + x = 6(b - x)$ and $a - x = 3(b + x)$. From the first equation, $a = 6b - 7x$, and, from the second equation, $a = 3b + 4x$. Therefore, $6b - 7x = 3b + 4x$, from which $b = \frac{11}{3}x$. Since a, b, and x are required to be positive integers, the smallest possible value for x is 3. Then $b = 11$ and $a = 6(11) - 7(3) = 45$. Therefore, 45 is the smallest number of pennies that Debbie could have had.

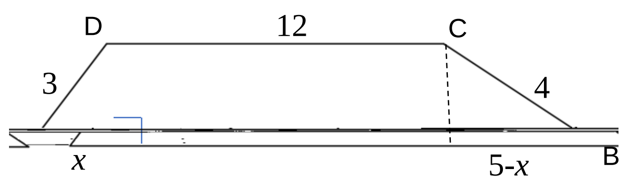
17. B Method 1:



In the diagram, construct a line through D parallel to leg BC. Quadrilateral DEBC is a parallelogram, making $\triangle ADE$ a 3-4-5 right triangle. The area of $\triangle ADE$ is $\frac{1}{2}(3)(4) = 6$. Thus, the altitude from D to AE has length $\frac{12}{5} = 2.4$. Therefore, the area of trapezoid ABCD is

$\frac{1}{2}(2.4)(12 + 17) = 34.8$

Method 2:



In the diagram, draw altitudes from D and C and represent the lengths of the segments as shown. Then, $h^2 = 9 - x^2 = 16 - (5 - x)^2$ from which $x = \frac{9}{5} = 1.8$. Thus, $h = \sqrt{9 - (1.8)^2} = 2.4$, and the area of trapezoid ABCD is $\frac{1}{2}(2.4)(12 + 17) = 34.8$

.

18. A The number of distinct arrangements of the nine digits is $\frac{9!}{(4!)(2!)(2!)}$ = 3780. A palindrome will have 4 in the fifth position and two 1's and one each of the 2's and 3's in positions 1 – 4. Positions 6 – 9 will mirror 1 – 4. The number of distinct ways of arranging these numbers is $\frac{4!}{2!}$ = 12. The probability that the number is a palindrome is

$\frac{12}{3780}$ $\frac{1}{315}$.

19. C Method 1:

Since DMA BNA, DAM BAN. Represent the measure of each as . Without loss of generality, let the length of the sides of the square be 2. Then DM = NB = 1 and AN = AM = $\sqrt{5}$, and cos $= \frac{2}{\sqrt{5}}$.

Sin (MAN) = sin(90 – 2) = cos2 = 2cos^2 – 1 = $\sqrt{\ }$ – .

Thus cos(MAN) = $\frac{4}{5}$ and tan(MAN) = $\frac{3}{4}$.

Method 2:

Let m(;= . Construct ‹‹‹$ ‹‹‹$ Then tan = $\frac{1}{2}$ and tan = 2.

Since m(;= – , tan(;= $\frac{r\_l' ? r\_l'}{5> r\_l' \; r\_l'}$ = $\frac{6?}{5>:6;@A}$ = $\frac{3}{4}$

20. D The coordinates of the midpoint of segment PQ are the average of the coordinates of P and Q, $\frac{x_1 \quad x_2}{2}, \frac{\log_a x_1 \quad \log_a x_2}{2}$. Since the line through the midpoint of segment PQ and point N is horizontal, the y-coordinate of point N is the same as the y-coordinate of the midpoint. Therefore, $y_3 \quad \log_a x_3 \quad \frac{\log_a x_1 \quad \log_a x_2}{2}$.

Using the laws of logarithms, $\log_a x_3 \quad \log_a (x_1)(x_2)^{\frac{1}{2}}$, from which $\sqrt{\quad}$.

21. E Let A, B, and C represent the number of students receiving each of the three grades. Then $\frac{5}{:}C \quad A \quad \frac{5}{9}B$ and $A + B \quad 23$. Since $A \quad \frac{5}{9}B$, $5A \quad B$, and $6A \quad B + A \quad 23$, then

4(r oran TEETOq0.00000912 0 612 792 reW*nBTF1 12 Tf1 0 0 1 212.93 102.51 032.5 g0 G[ )]TJTT56 T57 T580047 T480051 T

22. A Represent the diameter of the larger semicircle as $2a$, and the diameter of the smaller semicircle as $2b$. Note that the line joining the centers of the two semicircles passes through their common point. Using the Pythagorean Theorem,
$(2a - b)^2 = a^2 + (a + b)^2 \Rightarrow 4a^2 - 4ab + b^2 = a^2 + a^2 + 2ab + b^2$.
Simplifying, $4a^2 = 6ab \Rightarrow \frac{2b}{2a} = \frac{2}{3}$.

23. B We are looking for the solutions to the equation $x^2 + y - x - y^2 = 2018$.
Rewriting the left side as $x^2 - y^2 - (x - y)$ and factoring, the equation becomes $(x - y)(x + y - 1) = 2018$. Since $x \geq 1$ and $y \geq 1$, then $x + y - 1$ is positive, and thus