THE 2019 ±2020 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I ±MULTIPLE CHOICE

For [(1CHOr [(1Cu[(1Cu[(Q E(E)-2)11(t(E)-roM)-roMa5yC6 [(Ma5y Ma5yow)15(iMa5yng 26 [(5 ques)7t(E)-ri

6. Suppose that a and b are positive integers, c is a real number, and i is the imaginary unit.

If := E

13. If t { y$_{\tilde{O}}$ is a factor of y {t $_{\tilde{O}}$ (where t { y$_{\tilde{O}}$

20. The first term of an arithmetic sequence of distinct terms is 4. The 1st, 5th, 15th and kth terms of the arithmetic sequence form a geometric sequence in the same order. Compute the value of k.

(A) 25 (B) 35 (C) 36 (D) 39 (E) 40

21. For which of the following values of x will $\frac{:ë>5\ 4\ 7\ =;9}{ë}$ be an integer, while $\frac{ë>5\ 4\ 7\ =\ 9}{ë}$ is not?

(A) 2079 (B) 3575 (C) 5136 (D) 6237 (E) None of these

22. Let p, q, and r be the roots of the equation T7 F s vT6 E t { T F v L r. Compute the value of the expression $\left(1\ \frac{1}{p}\right)\left(1\ \frac{1}{q}\right)\left(1\ \frac{1}{r}\right)$.

(A) $\frac{7}{2}$ (B) $\frac{1}{4}$ (C) -3 (D) -12 (E) $\frac{29}{2}$

23. In triangle ABC, AB = 7, BC = 33, and AC = 37.
A circle centered at A with radius AB intersects
ray CB at point D and side AC at point E, as shown.
Compute the

Solutions

1. A Each of the three rows has four 1x2 rectangles. Each of the five columns has two 1x2 rectangles. There are a total of four 2x4 rectangles. The total is 26.

2. E Since 2 is the only even prime number, the smallest number in set A is 2 + 2019 = 2021. All the rest of the numbers in A are the sum of two odd numbers and are, therefore, even. The smallest number in set B is (2)(2019) = 4038, and all the rest of the numbers in B are odd. Thus A ˆ B is empty and the number of elements in the intersection is 0.

3. D 1 + 2 + 3 + D = 360 and 4 + 5 + E = 180.
Adding these two equations:
1 + 2 + 3 + 4 + 5 + D + E = 540. Also D + E = 180 ± x.
Therefore, 1 + 2 + 3 + 4 + 5 + 180 ± x = 540 from which
1 + 2 + 3 + 4 + 5 = 360 + x.

4. D $6^{2\ 3y}\ \frac{6^2}{6^{3y}}\ \frac{36}{6^{3y}}\ 2$. From this, 6^{3y} 18. Therefore, x^6>7ì L :x^6;:x^7ì ; L :u x;:s z; L x v z

5. B Let T = # of families with twins, R = # of families with triplets, and Q = # of families with quadruplets. Then we are given T + R + Q = 26 and 2T = 3R = 4Q. From the second equation
$T = \frac{3}{2}R$ and $Q = \frac{3}{4}R$. Therefore, $\frac{3}{2}R + R + \frac{3}{4}R = 26$ and R = 8.

6. B When := E >E^7 is expanded, the real part is =^7 F u=>^6. Therefore,
=^7 F u=>^6 L =:=^6 F u>^6; L F y v ä Since a and b are positive integers, and a is a factor of 74, a little trial and error gives a = 1 and b = 5 as the only solution, and a ± b

10. B Let the correct two-digit score be $10A + B$. Then the mis-entered score was $10B + A$.
Since the class average was 2.7 points less than it should have been, and there are 20
V W X G H Q W V L Q W K-H F O D V V $ E E \ ¶ V P L V V

.

24. B « $\frac{(48)(49)}{2} = (24)(49)$. Let n = the number of integers from A to B,

inclusive. Thus $n < 48$ and the sum of the integers from A to B, inclusive, is $\frac{n}{2}(A + B)$.

Therefore, $6\left[\frac{n}{2}(A + B)\right] = (24)(49)$ Ÿ $3n(A + B) =$:t ^{7};:u;:y^6; Ÿ $n(A + B) =$:t ^{7};:y^6;

Since $n < 48$, the only possible values of n are 2, 4, 7, 8, 14, and 28.

Letting n equal each of these values leads to the following results

$n = 2$ A